New Multidirectional Mean Value Inequality

M. Hamamdjiev, M. Ivanov

We establish a new type of multidirectional inequality, compare it with previously known results and discuss further work on the topic. The main result and its proof can be found in [1].

Let $B_\delta := B + \delta B_X$, where B is any subset of a Banach space X and B_X is the closed unit ball of X. For $A, B \subseteq X$ let $[A, B]$ be the convex hull of A and B.

Theorem 1 *Let X be a Banach space and let ∂ be a feasible subdifferential. Let A and B be non-empty closed, bounded and convex subsets of X. Let $f : X \to \mathbb{R} \cup \{\infty\}$ be a proper lower semicontinuous function such that $A \cap \operatorname{dom} f \neq \emptyset$. Let f be bounded below on $C := \overline{[A, B]_\delta}$ for some $\delta > 0$. Let*

$$\mu < \inf_C f.$$

Let $r, s \in \mathbb{R}$ be such that

$$r = \inf_A f, \;\; s < \inf_{B_\delta} f.$$

Then for each $\varepsilon > 0$ there are $\xi \in [A, B]_\delta$ and $p \in \partial f(\xi)$ such that

$$f(\xi) < \inf_{[A,B]} f + |r - s| + \varepsilon,$$

$$\|p\| < \frac{\max\{r, s\} - \mu}{\delta} + \varepsilon,$$

and

$$\inf_B p - \inf_A p > s - r.$$

References

[1] Mihail Hamamdjiev and Milen Ivanov, New Multirectional Mean Value Inequality, Journal of Convex Analysis, Volume 25, 2018, No. 4.